

Sudden death of entanglement and teleportation fidelity loss via the Unruh effect

André G. S. Landulfo* and George E. A. Matsas†

*Instituto de Física Teórica, Universidade Estadual Paulista,
Rua Dr. Bento Teobaldo Ferraz, 271 - Bl. II, 01140-070, São Paulo, SP, Brazil*

(Dated: July 2, 2009)

We use the Unruh effect to investigate how the teleportation of quantum states is affected when one of the entangled qubits used in the process is under the influence of some external force. In order to reach a comprehensive understanding, a detailed analysis of the acceleration effect on such entangled qubit system is performed. In particular, we calculate the mutual information and concurrence between the two qubits and show that the latter has a “sudden death” at a finite acceleration, whose value will depend on the time interval along which the detector is accelerated.

PACS numbers: 03.65.-w, 03.65.Ud, 04.62.+v

I. INTRODUCTION

The teleportation of quantum states is undoubtedly one of the most interesting effects unveiled in the last decade. In the original work by Bennett et al [1] the system is considered to be isolated from external forces and the maximally entangled qubit pair is unitarily evolved. As a natural development, Alsing and Milburn analyzed the case when the system is not quite isolated [2]. In their set up, Bob is replaced by a uniformly accelerated observer named Rob. Alice and Rob each hold an optical cavity at rest in their local frames, which are assumed to be initially free of Minkowski photons. Each cavity supports two orthogonal Minkowski modes A_i and R_i ($i = 1, 2$) with the same frequency, where hereafter A and R will be used to label Alice and Rob, respectively. At the moment that Alice and Rob overlap, they create an entangled pair

$$|0\rangle_M \otimes |0\rangle_M + |1\rangle_M \otimes |1\rangle_M \quad (1.1)$$

where

$$|0\rangle_M = |1\rangle_{X_1} \otimes |0\rangle_{X_2}, \quad |1\rangle_M = |0\rangle_{X_1} \otimes |1\rangle_{X_2}$$

and $X = A$ and R for the first and second qubits in Eq. (1.1), respectively. Then it is argued that as Rob is accelerated, his cavity would be populated by thermally excited Rindler photons as it would be predicted by the Unruh effect [3] (see also Ref. [4] for a recent review on the Unruh effect and its applications) and is concluded that the teleportation fidelity would be reduced. We note however, that the set up proposed above presents some conceptual difficulties [5]. In particular, the relationship between the Minkowski and Rindler modes as used in Ref. [2] is valid in the Minkowski spacetime without boundary conditions imposed by the presence of cavities.

In order to circumvent these difficulties, we introduce here a distinct set up which avoids the use of cavities.

For this purpose the qubits are modeled by a two-level semiclassical detector coupled to a massless scalar field. The detector is classical in the sense that it has a well defined worldline but quantum because of the nature of its internal degrees of freedom. The paper is organized as follows. In Sec. II we introduce our qubit and its interaction with the Klein-Gordon field. In Sec. III we entangle a pair of those qubits and use the Unruh effect to calculate its final state when one of them is uniformly accelerated for some fixed amount of proper time while the other one is inertial. Next, we investigate the corresponding mutual information and concurrence as a function of the non-inertial qubit acceleration. In particular, we verify that the qubit system experiences a sudden death of entanglement (see, e.g., Ref. [6, 7] and references therein) at a finite proper acceleration. In Sec. IV we revisit the original teleportation protocol [1] when the inertial Bob is replaced by the accelerated Rob and calculate how the teleportation fidelity diminishes as the acceleration grows. We dedicate Sec. V for our closing remarks and to establish a relationship between our theoretical model and a possible experimental physical set up. We adopt spacetime signature $(-+++)$ and assume natural units $c = \hbar = 1$ unless stated otherwise.

II. TWO-LEVEL DETECTOR QUBIT MODEL

We model our qubit in Minkowski spacetime (\mathbb{R}^4, g_{ab}) through a two-level detector with energy gap Ω as introduced by Unruh and Wald [8]. Here, we briefly revisit the corresponding detector theory and derive the necessary results for the forthcoming sections. The detector proper Hamiltonian is defined as

$$H_D = \Omega D^\dagger D, \quad (2.1)$$

where $D|0\rangle = D^\dagger|1\rangle = 0$, $D|1\rangle = |0\rangle$ and $D^\dagger|0\rangle = |1\rangle$, and $|0\rangle, |1\rangle$ are the corresponding unexcited, excited energy eigenstates, respectively. We couple the detector to a massless scalar field ϕ satisfying the Klein-Gordon equation [9]

$$\nabla_a \nabla^a \phi = 0 \quad (2.2)$$

*Electronic address: landulfo@ift.unesp.br

†Electronic address: matsas@ift.unesp.br

through the Hamiltonian

$$H_{\text{int}}(t) = \epsilon(t) \int_{\Sigma_t} d^3\mathbf{x} \sqrt{-g} \phi(x) [\psi(\mathbf{x})D + \bar{\psi}(\mathbf{x})D^\dagger], \quad (2.3)$$

where $\phi(x)$ is the free Klein-Gordon field operator, $g \equiv \det(g_{ab})$ and \mathbf{x} are coordinates defined on the Cauchy surface $\Sigma_{t=\text{const}}$ associated with some suitable timelike isometry. For our present purposes we assume that the detector follows either the inertial or the uniformly accelerated isometry of the Minkowski spacetime. Here $\epsilon \in C_0^\infty(\mathbb{R})$ is a smooth compact-support real-valued function, which keeps the detector switched on for a finite amount of proper time Δ (for more details on finite-time detectors see, e.g., Ref. [10]) and $\psi \in C_0^\infty(\Sigma_t)$ is a smooth compact-support complex-valued function, which models the fact that the detector only interacts with the field in a neighborhood of its worldline. The same detector model was recently used by Kok and Yurtsever to analyze the decoherence of an accelerated qubit due to the Unruh effect [11]. Using Eqs. (2.1)-(2.3) we cast the total Hamiltonian as

$$H_D \phi = H_0 + H_{\text{int}}, \quad (2.4)$$

where $H_0 = H_D + H_{KG}$ is the combined detector-field free Hamiltonian. In the interaction picture the state $|\Psi_t^{D\phi}\rangle$ describing the system at moment t can be written as

$$|\Psi_t^{D\phi}\rangle = T \exp[-i \int_{-\infty}^t dt' H_{\text{int}}^I(t')] |\Psi_{-\infty}^{D\phi}\rangle, \quad (2.5)$$

where T is the time-ordering operator and

$$H_{\text{int}}^I(t) = U_0^\dagger(t) H_{\text{int}}(t) U_0(t) \quad (2.6)$$

with $U_0(t)$ being the unitary evolution operator associated with $H_0(t)$. By using Eq. (2.5), we write $|\Psi_{\infty}^{D\phi}\rangle = |\Psi_{t>\Delta}^{D\phi}\rangle$ as

$$|\Psi_{\infty}^{D\phi}\rangle = T \exp[-i \int d^4x \sqrt{-g} \phi(x) (fD + \bar{f}D^\dagger)] |\Psi_{-\infty}^{D\phi}\rangle, \quad (2.7)$$

where $f \equiv \epsilon(t) e^{-i\Omega t} \psi(\mathbf{x})$ is a compact support complex function defined in Minkowski spacetime and we have used that $D^I = e^{-i\Omega t} D$. In first perturbation order, Eq. (2.7) becomes

$$|\Psi_{\infty}^{D\phi}\rangle = [I - i(\phi(f)D + \phi(f)^\dagger D^\dagger)] |\Psi_{-\infty}^{D\phi}\rangle, \quad (2.8)$$

where [12]

$$\begin{aligned} \phi(f) &\equiv \int d^4x \sqrt{-g} \phi(x) f \\ &= i[a(\overline{KEf}) - a^\dagger(KEf)] \end{aligned} \quad (2.9)$$

is an operator valued distribution obtained by smearing out the field operator by the testing function f above.

Here $a(\bar{u})$ and $a^\dagger(u)$ are annihilation and creation operators of u modes, respectively, the K operator takes the positive-frequency part of the solutions of Eq. (2.2) with respect to the timelike isometry, and

$$Ef = \int d^4x' \sqrt{-g(x')} [G^{\text{adv}}(x, x') - G^{\text{ret}}(x, x')] f(x'), \quad (2.10)$$

where G^{adv} and G^{ret} are the advanced and retarded Green functions, respectively. Next, by imposing that $\epsilon(t)$ is a very slow-varying function of time compared to the frequency Ω and that $\Delta \gg \Omega^{-1}$, we have that f is an approximately positive-frequency function, i.e., $KEf \approx Ef$ and $KE\bar{f} \approx 0$ (see appendix A). Now, by defining

$$\lambda \equiv -KEf, \quad (2.11)$$

we cast Eq. (2.9) as

$$\phi(f) \approx ia^\dagger(\lambda) \quad (2.12)$$

and Eq. (2.8) as

$$|\Psi_{\infty}^{D\phi}\rangle = (I + a^\dagger(\lambda)D - a(\bar{\lambda})D^\dagger) |\Psi_{-\infty}^{D\phi}\rangle. \quad (2.13)$$

The expression above carries the well known physical message that the excitation and deexcitation of an Unruh-DeWitt detector following a timelike isometry is associated with the absorption and emission, respectively, of a particle as “naturally” defined by observers comoving with the detector, i.e., in our case, Minkowski and Rindler particles for inertial and uniformly accelerated observers, respectively.

III. ENTANGLED QUBIT PAIR AND THE UNRUH EFFECT

Let us consider now a two-qubit system initially entangled as given by

$$|\Psi_{AR}\rangle = \alpha|0_A\rangle \otimes |1_R\rangle + \beta|1_A\rangle \otimes |0_R\rangle \quad (3.1)$$

with $|\alpha|^2 + |\beta|^2 = 1$, where $\{|0_X\rangle, |1_X\rangle\}$ is an orthonormal basis of the internal qubit space \mathfrak{H}_X and $X = A, R$. The free Hamiltonian for each one of the detectors is given by Eq. (2.1) with D replaced by A or R depending on the detector. Now, we impose that Alice’s detector is kept inertial in contrast to Rob’s one which is uniformly accelerated for a finite proper time Δ , having worldline

$$t(\tau) = a^{-1} \sinh a\tau, \quad x(\tau) = a^{-1} \cosh a\tau, \quad y(\tau) = z(\tau) = 0, \quad (3.2)$$

where τ and a are the detector’s proper time and acceleration, respectively, and here (t, x, y, z) are the usual Cartesian coordinates of Minkowski spacetime. The detectors are designed to be switched on only when they are accelerated. Thus, Alice’s inertial qubit only interacts with the scalar field indirectly through Rob’s detector.

At the end of the paper we discuss a laboratory situation which realizes these assumptions.

Rob's qubit interacts with the field according to the Hamiltonian (2.3) with the proper replacements: $D \rightarrow R$ and $t \rightarrow \tau$, where Σ_τ are spacelike hypersurfaces orthogonal to the congruence of boost isometries to which Rob's detector worldline belongs. The total Hamiltonian is given by

$$H_{AR\phi} = H_A + H_R + H_{KG} + H_{\text{int}}. \quad (3.3)$$

The corresponding Hilbert space associated with our system can be written now as $\mathfrak{H}_T = \mathfrak{H}_A \otimes \mathfrak{H}_R \otimes \mathfrak{F}_s(\mathfrak{H}_I \oplus \mathfrak{H}_{II})$, where $\mathfrak{F}_s(\mathfrak{H}_I \oplus \mathfrak{H}_{II})$ is the symmetric Fock space of $\mathfrak{H}_I \oplus \mathfrak{H}_{II}$ with \mathfrak{H}_I being the Hilbert space of positive-frequency solutions with respect to τ with initial data on Σ_I which is the portion of $\Sigma_{\tau=0}$ in the right Rindler wedge defined by $x > |t|$, and analogously for \mathfrak{H}_{II} and the left Rindler wedge defined by $x < -|t|$.

Next by using the fact that Rob's detector is the only one which interacts with the field and that this is confined in the right Rindler wedge, we use Eq. (2.13) to evolve our initial state

$$|\Psi_{-\infty}^{AR\phi}\rangle = |\Psi_{AR}\rangle \otimes |0_M\rangle, \quad (3.4)$$

with $|0_M\rangle$ being the Minkowski vacuum (i.e., the no-particle state as defined by inertial observers), to its asymptotic form

$$|\Psi_{\infty}^{AR\phi}\rangle = (I + a_{RI}^\dagger(\lambda)R - a_{RI}(\bar{\lambda})R^\dagger)|\Psi_{-\infty}^{AR\phi}\rangle, \quad (3.5)$$

where the labels in a_{RI}^\dagger and a_{RI} emphasize that they are creation and annihilation operators of Rindler modes in the right wedge (I), $\lambda = -KEf \approx Ef$, and here $f = \epsilon(\tau)e^{-i\Omega\tau}\psi(\mathbf{x})$. By using Eqs. (3.1) and (3.4) in Eq. (3.5), we obtain

$$\begin{aligned} |\Psi_{\infty}^{AR\phi}\rangle &= |\Psi_{-\infty}^{AR\phi}\rangle + \alpha|0_A\rangle \otimes |0_R\rangle \otimes (a_{RI}^\dagger(\lambda)|0_M\rangle) \\ &+ \beta|1_A\rangle \otimes |1_R\rangle \otimes (a_{RI}(\bar{\lambda})|0_M\rangle). \end{aligned} \quad (3.6)$$

In order to proceed, we write a_{RI} and a_{RI}^\dagger in terms of the annihilation, a_M , and creation, a_M^\dagger , operators of Minkowski modes as [8]

$$a_{RI}(\bar{\lambda}) = \frac{a_M(\bar{F}_{1\Omega}) + e^{-\pi\Omega/a}a_M^\dagger(F_{2\Omega})}{(1 - e^{-2\pi\Omega/a})^{1/2}}, \quad (3.7)$$

$$a_{RI}^\dagger(\lambda) = \frac{a_M^\dagger(F_{1\Omega}) + e^{-\pi\Omega/a}a_M(\bar{F}_{2\Omega})}{(1 - e^{-2\pi\Omega/a})^{1/2}}, \quad (3.8)$$

where

$$F_{1\Omega} = \frac{\lambda + e^{-\pi\Omega/a}\lambda \circ w}{(1 - e^{-2\pi\Omega/a})^{1/2}}, \quad (3.9)$$

$$F_{2\Omega} = \frac{\bar{\lambda} \circ w + e^{-\pi\Omega/a}\bar{\lambda}}{(1 - e^{-2\pi\Omega/a})^{1/2}}, \quad (3.10)$$

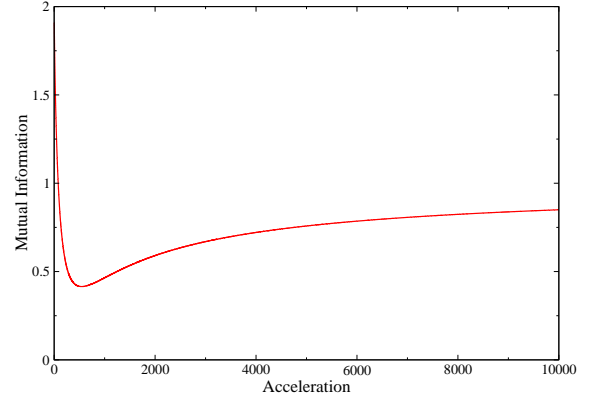


FIG. 1: The graph exhibits the mutual information $I(A : R)$ for a singlet initial state as a function of acceleration a/Ω , with $\epsilon^2 = 8\pi^2 \cdot 10^{-6}$, $\Omega = 100$, $\Delta = 1000$ and $\kappa = 0.02$. The most interesting feature is related with the fact that the curve is not monotonic, acquiring its minimum value at $a_0/\Omega \approx 545.75$. We note that the dimensionless quantity a/Ω reflects the temperature of the Unruh thermal bath as experienced by Rob's detector per its energy gap (up to a $1/(2\pi)$ factor).

$w(t, x, y, z) = (-t, -x, y, z)$ is the wedge reflection isometry, and we recall that whenever $\varphi \in \mathfrak{H}_I$ then $\varphi \circ w \in \mathfrak{H}_{II}$. For further convenience, let us define from Eq. (2.11)

$$\nu^2 \equiv \|\lambda\|^2, \quad (3.11)$$

where

$$(F_{i\Omega}, F_{j\Omega})_{KG} = \|\lambda\|^2 \delta_{ij}, \quad i \in \{1, 2\}. \quad (3.12)$$

Here we write the Klein-Gordon internal product

$$(F_{i\Omega}, F_{j\Omega})_{KG} \equiv i \int_{\Sigma} d^3x \sqrt{h} (\bar{F}_{i\Omega} \nabla_a F_{j\Omega} - (\nabla_a \bar{F}_{i\Omega}) F_{j\Omega}) n^a$$

between the positive-frequency solutions, $F_{i\Omega}$ and $F_{j\Omega}$, with respect to the Minkowski time t taken on a Cauchy surface Σ with unit orthogonal vector n^a and $h \equiv \det(h_{ab})$ with h_{ab} being the restriction of g_{ab} on Σ . By assuming our detector to be localized as given by the Gaussian $\psi(\mathbf{x}) = (\kappa\sqrt{2\pi})^{-3} \exp(-\mathbf{x}^2/2\kappa^2)$ with variance $\kappa = \text{const} \ll 1$, we show in the appendix A that

$$\nu^2 = \frac{\epsilon^2 \Omega \Delta}{2\pi} e^{-\Omega^2 \kappa^2}. \quad (3.13)$$

Now, by using Eqs. (3.7) and (3.8) to write

$$a_{RI}(\bar{\lambda})|0_M\rangle = \frac{\nu e^{-\pi\Omega/a}}{(1 - e^{-2\pi\Omega/a})^{1/2}} |1_{\bar{F}_{2\Omega}}\rangle, \quad (3.14)$$

$$a_{RI}^\dagger(\lambda)|0_M\rangle = \frac{\nu}{(1 - e^{-2\pi\Omega/a})^{1/2}} |1_{F_{1\Omega}}\rangle, \quad (3.15)$$

we cast Eq. (3.6) in the form

$$\begin{aligned} |\Psi_{\infty}^{AR\phi}\rangle &= |\Psi_{-\infty}^{AR\phi}\rangle + \alpha \nu \frac{|0_A\rangle \otimes |0_R\rangle \otimes |1_{F_{1\Omega}}\rangle}{(1 - e^{-2\pi\Omega/a})^{1/2}} \\ &+ \beta \nu e^{-\pi\Omega/a} \frac{|1_A\rangle \otimes |1_R\rangle \otimes |1_{\bar{F}_{2\Omega}}\rangle}{(1 - e^{-2\pi\Omega/a})^{1/2}}, \end{aligned} \quad (3.16)$$

where $\tilde{F}_{i\Omega} = F_{i\Omega}/\nu$. Notice that the fact that every Rob's qubit transition demands the emission of a Minkowski particle is codified in Eq. (3.16).

The density matrix which describes the two-qubit state is obtained tracing out the scalar field degrees of freedom, namely,

$$\rho_{\infty}^{AR} = ||\Psi_{\infty}^{AR\phi}||^{-2} \text{tr}_{\phi} |\Psi_{\infty}^{AR\phi}\rangle \langle \Psi_{\infty}^{AR\phi}|, \quad (3.17)$$

where

$$||\Psi_{\infty}^{AR\phi}||^2 = 1 + \frac{|\alpha|^2 \nu^2}{1 - e^{-2\pi\Omega/a}} + \frac{|\beta|^2 \nu^2 e^{-2\pi\Omega/a}}{1 - e^{-2\pi\Omega/a}}$$

normalizes the final density matrix, i.e., $\text{tr} \rho_{\infty}^{AR} = 1$. By working out Eq. (3.17), we obtain

$$\begin{aligned} \rho_{\infty}^{AR} = & 2S_0^{\alpha\beta} |\Psi_{AR}\rangle \langle \Psi_{AR}| + S_2^{\alpha\beta} |0_A\rangle \otimes |0_R\rangle \langle 0_A| \otimes \langle 0_R| \\ & + S_1^{\alpha\beta} |1_A\rangle \otimes |1_R\rangle \langle 1_A| \otimes \langle 1_R|, \end{aligned} \quad (3.18)$$

where

$$\begin{aligned} S_0^{\alpha\beta} &= \frac{(1 - e^{-2\pi\Omega/a})/2}{(1 - e^{-2\pi\Omega/a}) + |\alpha|^2 \nu^2 + |\beta|^2 \nu^2 e^{-2\pi\Omega/a}}, \\ S_1^{\alpha\beta} &= \frac{|\beta|^2 \nu^2 e^{-2\pi\Omega/a}}{(1 - e^{-2\pi\Omega/a}) + |\alpha|^2 \nu^2 + |\beta|^2 \nu^2 e^{-2\pi\Omega/a}}, \\ S_2^{\alpha\beta} &= \frac{|\alpha|^2 \nu^2}{(1 - e^{-2\pi\Omega/a}) + |\alpha|^2 \nu^2 + |\beta|^2 \nu^2 e^{-2\pi\Omega/a}}, \end{aligned}$$

and we verify that $2S_0^{\alpha\beta} + S_1^{\alpha\beta} + S_2^{\alpha\beta} = 1$. For the sake of convenience, we cast Eq. (3.18) in matrix form as

$$\rho_{\infty}^{AR} = \begin{pmatrix} S_2^{\alpha\beta} & 0 & 0 & 0 \\ 0 & 2|\alpha|^2 S_0^{\alpha\beta} & 2\alpha\bar{\beta} S_0^{\alpha\beta} & 0 \\ 0 & 2\bar{\alpha}\beta S_0^{\alpha\beta} & 2|\beta|^2 S_0^{\alpha\beta} & 0 \\ 0 & 0 & 0 & S_1^{\alpha\beta} \end{pmatrix}, \quad (3.19)$$

where we have used the basis

$$\{|0_A\rangle \otimes |0_R\rangle, |0_A\rangle \otimes |1_R\rangle, |1_A\rangle \otimes |0_R\rangle, |1_A\rangle \otimes |1_R\rangle\}.$$

A. Mutual information

In order to extract information on the correlation between the qubits A and R , we calculate the mutual information [13, 14]

$$I(A : R) = S(\rho_{\infty}^A) + S(\rho_{\infty}^R) - S(\rho_{\infty}^{AR}), \quad (3.20)$$

where $0 \leq I(A : R) \leq 2$. Here $\rho_{\infty}^A = \text{tr}_R \rho_{\infty}^{AR}$, $\rho_{\infty}^R = \text{tr}_A \rho_{\infty}^{AR}$, and $S(\rho) = -\text{tr}(\rho \log_2 \rho)$ is the von Neumann entropy. In Fig. 1 we plot the mutual information for a fixed proper time interval Δ along which Rob's detector is accelerated, assuming the two-qubit system to be initially in a singlet state: $\alpha = -\beta = 1/\sqrt{2}$. We see that for low enough accelerations, the mutual information keeps

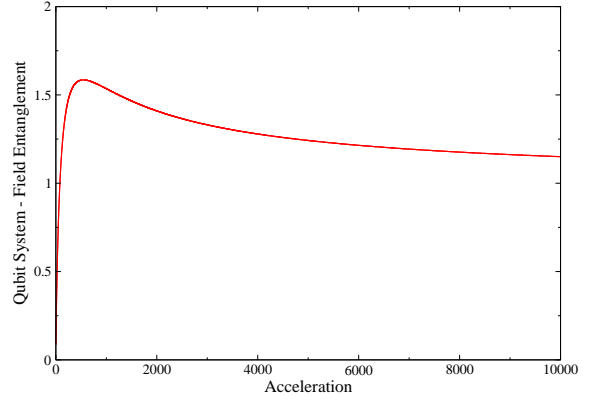


FIG. 2: The graph shows the entanglement, $E^{AR\phi}$, between the qubit system and field as a function of acceleration a/Ω assuming the same initial state and ϵ , Ω , Δ , and κ parameters as in Fig. 1. We note that the entanglement takes its maximum value, $E_{\max}^{AR\phi} \approx 1.58$, at $a_0/\Omega \approx 545.75$, precisely where $I(A : R)$ has its minimum. This is interesting to note that because the normalized $|\Psi_{\infty}^{AR\phi}\rangle$ is Schmidt decomposed [13] [see Eq. (3.16)], the corresponding Schmidt number is 3 and the maximum entanglement $E_{\max}^{AR\phi} = \log 3 \approx 1.58$.

its value close to the maximum one, $I(A : R) \approx 2$, as expected. This is so because for very low accelerations the temperature of the Unruh thermal bath is small containing, thus, quite few particles with proper energy Ω able to interact with the detector. The reason why $I(A : R) \neq 2$ for arbitrarily small a is because even inertial detectors have a non-zero probability of spontaneously decaying with the emission of a Minkowski particle, which carries away information from the qubit-system. For arbitrarily large accelerations, where the detector experiences high Unruh temperatures, we have $I(A : R) \rightarrow 1$, indicating that the qubits are still correlated but not entangled, as it can be seen directly from Eq. (3.18):

$$\rho_{\infty}^{AR} \xrightarrow{a \rightarrow \infty} \frac{1}{2} |0_A\rangle \otimes |0_R\rangle \langle 0_A| \otimes \langle 0_R| + \frac{1}{2} |1_A\rangle \otimes |1_R\rangle \langle 1_A| \otimes \langle 1_R|.$$

In order to get a better understanding of the physical content codified in Fig. 1, this is interesting to analyze the entanglement between the two-qubit system and the field. Since $|\Psi_{\infty}^{AR\phi}\rangle$ is a pure state, the entanglement between the qubits and the field is given by [13]

$$E^{AR\phi} = S(\rho_{\infty}^{AR}) = S(\rho_{\infty}^{\phi}), \quad (3.21)$$

where ρ_{∞}^{AR} was defined in Eq. (3.17) and ρ_{∞}^{ϕ} is the density matrix obtained analogously by taking the partial trace on the qubits degrees of freedom. In Fig. 2 we plot the qubit system-field entanglement for the situation described in Fig. 1. The qubit system-field entanglement $E^{AR\phi}$ is small for low enough accelerations, since $|\Psi_{\infty}^{AR\phi}\rangle$ is approximately separable (but not exactly separable because again of the non-zero probability of spontaneous deexcitation of inertial detectors) in contrast to

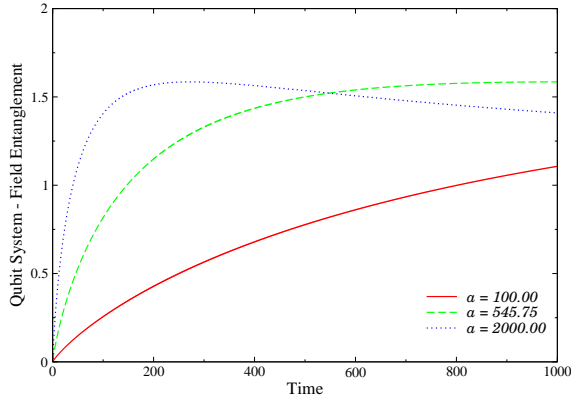


FIG. 3: The graph follows the behavior of the entanglement, $E^{AR\phi}$, along the time for three distinct proper accelerations: $a/\Omega = 100$ (full line), $a/\Omega = a_0 = 545.75$ (dashed line) and $a/\Omega = 2000$ (dotted line) assuming the same initial state and ϵ , Ω , Δ , and κ parameters as in Fig. 1. We note that for $a \leq a_0$, $E^{AR\phi}$ increases monotonically as a function of time. However, for $a > a_0$, the qubit system and field get maximally entangled at some time $\tau = \tau_e < \Delta$, after which the qubit system recovers back part of its correlations from the total system. Although not being visually evident, the graph is plotted in the acceleration time interval $\tau = [1, \Delta]$, which respects the constraint $\Omega\tau \gg 1$ [see discussion above Eq. (2.11)].

the case of arbitrarily large accelerations where $E^{AR\phi}$ approaches the unity. As for the mutual information, the qubit system-field entanglement has a non-trivial behavior acquiring its maximum value at $a = a_0$, which is precisely where $I(A : R)$ has its minimum (see Fig. 1). For $a \geq a_0$, the qubit system recovers part of its correlations after some time $\tau = \tau_e \leq \Delta$ as shown in Fig. 3.

B. Concurrence

Now, we show that the qubit-system entanglement experiences a sudden death for accelerations smaller than the one necessary for the mutual information to acquire its minimum. For this purpose, we calculate the concurrence [15]

$$C(\rho_{\infty}^{AR}) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (3.22)$$

associated with our mixed state ρ_{∞}^{AR} , where λ_i ($i = 1, \dots, 4$) are the eigenvalues of $\rho_{\infty}^{AR}(\sigma_y \otimes \sigma_y) \bar{\rho}_{\infty}^{AR}(\sigma_y \otimes \sigma_y)$ with $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ and $\bar{\rho}_{\infty}^{AR}$ is obtained by taking the complex conjugate of every term in Eq. (3.19). In Fig. 4 we see that for arbitrarily small a , the qubit system has $C(\rho_{\infty}^{AR}) \approx 1$ which is in agreement with $I(A : R) \approx 2$ found in the low acceleration regime. Now, as the acceleration increases the entanglement between the qubits decreases monotonically vanishing at a definite value

$$a/\Omega = a_{sd}/\Omega = \pi / \ln(\nu^2/2 + \sqrt{1 + \nu^4/4}).$$

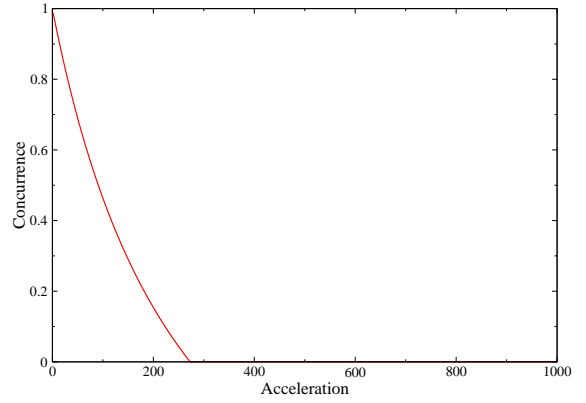


FIG. 4: The concurrence $C(\rho_{\infty}^{AR})$ is plotted as a function of the acceleration a/Ω assuming the same initial state and ϵ , Ω , Δ , and κ parameters as in Fig. 1. The sudden death of the entanglement between the two qubits is observed at $a_{sd}/\Omega \approx 273.00$.

Thus for a fixed acceleration time interval Δ , the two qubits lose their entanglement for every acceleration $a \geq a_{sd}$.

IV. TELEPORTATION AND THE UNRUH EFFECT

Now, let us use our previous results to revisit the teleportation protocol when Alice and Rob initially share the entangled qubit system (3.1) in a singlet state, $\alpha = -\beta = 1/\sqrt{2}$, and calculate how the corresponding fidelity is affected as a function of Rob's qubit acceleration. The state to be teleported by Alice is given by

$$|\varphi_C\rangle = \gamma|0_C\rangle + \delta|1_C\rangle, \quad (4.1)$$

which combined with $|\Psi_{AR}\rangle$ given in Eq. (3.1) and the Minkowski vacuum lead to the following total initial state

$$|\Psi_{-\infty}^{CAR\phi}\rangle = |\varphi_C\rangle \otimes |\Psi_{AR}\rangle \otimes |0_M\rangle. \quad (4.2)$$

By using now that

$$\begin{aligned} |0_C\rangle \otimes |0_A\rangle &= \frac{1}{\sqrt{2}}(|\phi_{CA}^+\rangle + |\phi_{CA}^-\rangle), \\ |0_C\rangle \otimes |1_A\rangle &= \frac{1}{\sqrt{2}}(|\psi_{CA}^+\rangle + |\psi_{CA}^-\rangle), \\ |1_C\rangle \otimes |0_A\rangle &= \frac{1}{\sqrt{2}}(|\psi_{CA}^+\rangle - |\psi_{CA}^-\rangle), \\ |1_C\rangle \otimes |1_A\rangle &= \frac{1}{\sqrt{2}}(|\phi_{CA}^+\rangle - |\phi_{CA}^-\rangle), \end{aligned} \quad (4.3)$$

where $|\phi_{CA}^+\rangle, |\phi_{CA}^-\rangle, |\psi_{CA}^+\rangle, |\psi_{CA}^-\rangle$ are the Bell states [13], we cast Eq. (4.2) as

$$\begin{aligned} |\Psi_{-\infty}^{CAR\phi}\rangle = & \frac{1}{2} [|\phi_{CA}^+\rangle \otimes (\gamma|1_R\rangle - \delta|0_R\rangle) \otimes |0_M\rangle \\ & + |\phi_{CA}^-\rangle \otimes (\gamma|1_R\rangle + \delta|0_R\rangle) \otimes |0_M\rangle \\ & + |\psi_{CA}^+\rangle \otimes (-\gamma|0_R\rangle + \delta|1_R\rangle) \otimes |0_M\rangle \\ & - |\psi_{CA}^-\rangle \otimes (\gamma|0_R\rangle + \delta|1_R\rangle) \otimes |0_M\rangle]. \end{aligned}$$

The asymptotic total final state after Rob has accelerated for proper time Δ can be cast from Eq. (3.5) as

$$|\Psi_{\infty}^{CAR\phi}\rangle = (I + a_{RI}^{\dagger}(\lambda)R - a_{RI}(\bar{\lambda})R^{\dagger})|\Psi_{-\infty}^{CAR\phi}\rangle.$$

For the sake of simplicity, let us assume that Alice makes a Bell measurement obtaining $|\psi_{CA}^-\rangle$, which will be eventually informed to Rob by classical means. Then, we have

$$\begin{aligned} |\Psi_{\infty}^{CAR\phi}\rangle = & -\frac{1}{2}|\psi_{CA}^-\rangle \otimes (\gamma|0_R\rangle + \delta|1_R\rangle) \otimes |0_M\rangle \\ & + \frac{\gamma}{2}|\psi_{CA}^-\rangle \otimes |1_R\rangle \otimes a_{RI}(\bar{\lambda})|0_M\rangle \\ & - \frac{\delta}{2}|\psi_{CA}^-\rangle \otimes |0_R\rangle \otimes a_{RI}^{\dagger}(\lambda)|0_M\rangle \\ = & -\frac{1}{2}|\psi_{CA}^-\rangle \otimes (\gamma|0_R\rangle + \delta|1_R\rangle) \otimes |0_M\rangle \\ & + \frac{\nu\gamma e^{-\pi\Omega/a}}{2(1 - e^{-2\pi\Omega/a})^{1/2}}|\psi_{CA}^-\rangle \otimes |1_R\rangle \otimes |1_{\tilde{F}_{2\Omega}}\rangle \\ & - \frac{\nu\delta}{2(1 - e^{-2\pi\Omega/a})^{1/2}}|\psi_{CA}^-\rangle \otimes |0_R\rangle \otimes |1_{\tilde{F}_{1\Omega}}\rangle. \end{aligned}$$

The density matrix associated with Rob's qubit is

$$\rho_{\infty}^R = ||\Psi_{\infty}^{CAR\phi}||^{-2} \text{tr}_{\phi_{CA}} |\Psi_{\infty}^{CAR\phi}\rangle \langle \Psi_{\infty}^{CAR\phi}|, \quad (4.4)$$

where

$$||\Psi_{\infty}^{CAR\phi}||^2 = \frac{1}{4} \left(1 + \frac{|\gamma|^2 \nu^2 e^{-2\pi\Omega/a}}{1 - e^{-2\pi\Omega/a}} + \frac{|\delta|^2 \nu^2}{1 - e^{-2\pi\Omega/a}} \right). \quad (4.5)$$

Eq. (4.4) can be recast as

$$\begin{aligned} \rho_{\infty}^R = & (|\gamma|^2 S_0^{\gamma\delta} + |\delta|^2 S_2^{\gamma\delta})|0_R\rangle\langle 0_R| + \gamma\bar{\delta} S_0^{\gamma\delta}|0_R\rangle\langle 1_R| \\ & + \bar{\gamma}\delta S_0^{\gamma\delta}|1_R\rangle\langle 0_R| + (|\delta|^2 S_0^{\gamma\delta} + |\gamma|^2 S_1^{\gamma\delta})|1_R\rangle\langle 1_R|, \end{aligned} \quad (4.6)$$

where

$$S_0^{\gamma\delta} = \frac{1 - e^{-2\pi\Omega/a}}{1 - e^{-2\pi\Omega/a} + \nu^2|\delta|^2 + \nu^2|\gamma|^2 e^{-2\pi\Omega/a}}, \quad (4.7)$$

$$S_1^{\gamma\delta} = \frac{\nu^2 e^{-2\pi\Omega/a}}{1 - e^{-2\pi\Omega/a} + \nu^2|\delta|^2 + \nu^2|\gamma|^2 e^{-2\pi\Omega/a}}, \quad (4.8)$$

$$S_2^{\gamma\delta} = \frac{\nu^2}{1 - e^{-2\pi\Omega/a} + \nu^2|\delta|^2 + \nu^2|\gamma|^2 e^{-2\pi\Omega/a}}. \quad (4.9)$$

Let us choose $\gamma = \delta = 1/\sqrt{2}$ in Eq. (4.1). In this case, using the basis $\{|0_R\rangle, |1_R\rangle\}$ we have

$$\rho_{\infty}^R = \begin{pmatrix} S_0 + S_2 & S_0 \\ S_0 & S_0 + S_1 \end{pmatrix}, \quad (4.10)$$

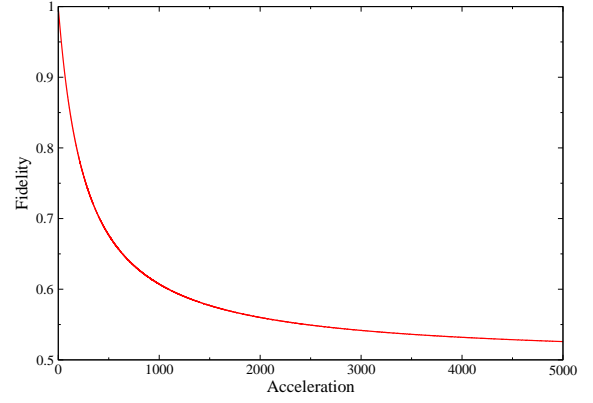


FIG. 5: The teleportation fidelity F is plotted as a function of the acceleration a/Ω with the values of ϵ , Ω , Δ and κ being the same as in Fig. 1.

with

$$S_0 \equiv \frac{S_0^{1/\sqrt{2} \ 1/\sqrt{2}}}{2}, \quad S_1 \equiv \frac{S_1^{1/\sqrt{2} \ 1/\sqrt{2}}}{2}, \quad S_2 \equiv \frac{S_2^{1/\sqrt{2} \ 1/\sqrt{2}}}{2}.$$

Finally, the teleportation fidelity $F \equiv \langle \varphi_C | \rho_{\infty}^R | \varphi_C \rangle$ turns out to be

$$F = S_0 + 1/2, \quad (4.11)$$

which is plotted in Fig. 5 as a function of Rob's qubit proper acceleration. We see from Fig 5 that for low enough accelerations $F \approx 1$ and for arbitrarily large accelerations $F \approx 0.5$. This is so because of the entanglement loss between Alice and Rob's qubits as discussed in the previous section. In contrast to Figs. 1 and 2 we see that F has a monotonous decrease as a function of a/Ω .

V. FINAL REMARKS

Technological developments have recently provided means to new and exquisite tests of quantum mechanics. This is not only interesting in connection with information theory but also with a number of conceptual issues. In particular the interplay between quantum mechanics and relativity has been a permanent source of preoccupation [16] which culminates with the long standing quest for quantum gravity. However, very interesting physics involving quantum mechanics and relativity can be already witnessed in Minkowski spacetime as, e.g., the fact that spin entanglement and entropy are not invariant by Lorentz transformations when the associated particles are described by wave packets [17, 18]. A consequence coming out from these facts is that Bell inequalities can be *satisfied* rather than violated if the spin detectors move fast enough [19]. In the present paper, we have analyzed how the teleportation fidelity is affected when one of the entangled qubits is uniformly accelerated for a finite time interval under the influence of some external agent. We

model our qubit to interact with a massless scalar field as it accelerates. An hypothetical laboratory realization of our model can be envisaged by using as qubit a charged fermion accelerated by an electric field pointing in the same direction of some background magnetic field along which the fermion spin is prepared [7]. The coupling between the spin and magnetic field gives rise to the qubit internal energy gap. Then, the unexcited and excited qubit states correspond to the cases where the spin points in the same and opposite directions with respect to the magnetic field, respectively. We have shown that the teleportation fidelity steadily decays as the acceleration increases for a fixed interaction proper time (see Fig. 5). From the point of view of inertial observers this is due to the fact that part of the entanglement between the qubits is carried away by the scalar radiation which is emitted when the accelerated qubit suffers a transition. This is confirmed by the fact that the *qubit-system mutual information* and the *qubit system-field entanglement* have a complementary behavior as a function of the acceleration magnitude, i.e. one decreases (increases) as the other one increases (decreases) (see Figs. 1 and 2). The non-triviality of these graphs, codified by the fact that the lines do not have a monotonous behavior can be understood from Fig. 3, which shows that after some long enough time τ_e the entanglement between the qubit system and field begins to decrease back. This is obvious for the case $a = 2000$. For $a \leq 545.75$ this behavior would also be seen if Δ were large enough. Remarkably, the concurrence which measures the entanglement of the qubit system experiences a sudden death for some acceleration a_{sd} as shown in Fig. 4. Finally, this is in order to call attention that from the point of view of uniformly accelerated observers the interpretation for the above results is quite different from the one due to inertial observers, since from their point of view the uniformly accelerated qubit interacts with the very Unruh thermal bath of real (Rindler) particles in which it is immersed in its proper frame. This is another example of how inertial and accelerated observers can give quite different physical interpretations concerning the same physical phenomenon although they must of course agree on the output measured by a given experimental set up (see, e.g., Ref. [20]).

Acknowledgments

A.L. and G.M. acknowledge full and partial financial support from Fundação de Amparo à Pesquisa do Estado de São Paulo, respectively. G.M. Also acknowledges partial support from Conselho Nacional de Desenvolvimento Científico e Tecnológico.

APPENDIX A: DERIVATION OF EQ. (3.13)

Here we calculate the ν coefficient introduced in Eq. (3.13). For this purpose, let us consider a general

smooth compact support function $f \in C_0^\infty(M)$ defined in a globally hyperbolic time-orientable spacetime (M, g_{ab}) and choose a Cauchy surface $\Sigma \subset M - J^+(\text{supp } f)$ outside the causal future of its support [21]. Now, let us define

$$\lambda(x) \equiv \int_M d^4x' \sqrt{-g(x')} G^{\text{adv}}(x, x') f(x'). \quad (\text{A1})$$

Then, $(\nabla^a \nabla_a - m^2)\lambda = f$ and we note that $\text{supp } \lambda \subset J^-(\text{supp } f)$. Hence, assuming $\phi \in C^\infty(M)$ to be any solution of Eq. (2.2), we have

$$\begin{aligned} \int_M d^4x \sqrt{-g} \phi f &= \int_{J^+(\Sigma)} d^4x \sqrt{-g} \phi f \\ &= \int_{J^+(\Sigma)} d^4x \sqrt{-g} \phi (\nabla^a \nabla_a - m^2)\lambda \\ &= \int_{J^+(\Sigma)} d^4x \sqrt{-g} \nabla^a (\phi \nabla_a \lambda - \lambda \nabla_a \phi) \\ &\quad + \int_{J^+(\Sigma)} d^4x \sqrt{-g} \lambda (\nabla^a \nabla_a - m^2)\phi \\ &= \int_\Sigma d^3x \sqrt{h} (\phi \nabla_a \lambda - \lambda \nabla_a \phi) n^a, \end{aligned}$$

where n^a is a unit normal vector orthogonal to Σ . Now, by using Eq. (2.10), we see that $Ef|_\Sigma = \lambda|_\Sigma$ and thus

$$\int_M d^4x \sqrt{-g} \phi f = \int_\Sigma d^3x \sqrt{h} (\phi \nabla_a (Ef) - (Ef) \nabla_a \phi) n^a. \quad (\text{A2})$$

Next, let us decompose Ef in terms of positive- and negative-frequency Rindler modes $u_{\omega \mathbf{k}_\perp}$ and $\bar{u}_{\omega \mathbf{k}_\perp}$, respectively, as

$$\begin{aligned} Ef &= \int_0^\infty d\omega \int d\mathbf{k}_\perp [(u_{\omega \mathbf{k}_\perp}, Ef)_{KG} u_{\omega \mathbf{k}_\perp} \\ &\quad - (\bar{u}_{\omega \mathbf{k}_\perp}, Ef)_{KG} \bar{u}_{\omega \mathbf{k}_\perp}], \end{aligned} \quad (\text{A3})$$

where $u_{\omega \mathbf{k}_\perp}$ satisfies $\nabla_a \nabla^a u_{\omega \mathbf{k}_\perp} = 0$ with $\mathbf{k}_\perp \equiv (k_y, k_z)$ and is eigenstate of $i\partial_\tau$, $-i\partial_y$ and $-i\partial_z$ with eigenvalues ω , k_y and k_z , respectively. Then, from Eq. (A2) we have

$$(u_{\omega \mathbf{k}_\perp}, Ef)_{KG} = i \int_M d^4x \sqrt{-g} f \bar{u}_{\omega \mathbf{k}_\perp}, \quad (\text{A4})$$

$$(\bar{u}_{\omega \mathbf{k}_\perp}, Ef)_{KG} = i \int_M d^4x \sqrt{-g} f u_{\omega \mathbf{k}_\perp}. \quad (\text{A5})$$

Let us now show that Eq. (A5) vanishes. For this purpose, we write $u_{\omega \mathbf{k}_\perp} = e^{-i\omega\tau} \varphi_{\omega \mathbf{k}_\perp}(\xi, \mathbf{x}_\perp)$, where

$$\varphi_{\omega \mathbf{k}_\perp}(\xi, \mathbf{x}_\perp) = \left[\frac{\sinh(\pi\omega/a)}{4\pi^4 a} \right]^{1/2} K_{i\omega/a}(k_\perp e^{a\xi}/a) e^{i\mathbf{k}_\perp \cdot \mathbf{x}_\perp}$$

with $K_\mu(z)$ being the modified Bessel function, $\mathbf{x}_\perp \equiv (y, z)$ and we are covering the right Rindler wedge with coordinates $(\tau, \xi, \mathbf{x}_\perp)$ in which case the corresponding line element becomes

$$ds^2 = e^{2a\xi} (-d\tau^2 + d\xi^2) + d\mathbf{x}_\perp^2.$$

Then, we integrate Eq. (A5) in the τ variable by using $\epsilon(\tau) \approx \epsilon = \text{const}$ when the detector is switched on (and $\epsilon(\tau) = 0$ when the detector is switched off), obtaining

$$(\bar{u}_{\omega\mathbf{k}_\perp}, Ef)_{KG} = 2i\epsilon\gamma_{\omega\mathbf{k}_\perp} \frac{\sin[(\omega + \Omega)\Delta/2]}{(\omega + \Omega)}, \quad (\text{A6})$$

where $\gamma_{\omega\mathbf{k}_\perp} \equiv \int_\Sigma d^3x \sqrt{-g} \psi(\mathbf{x}) \varphi_{\omega\mathbf{k}_\perp}$. Then, by using the fact that

$$\frac{\sin[(\omega + \Omega)\Delta/2]}{(\omega + \Omega)} \approx \pi\delta(\omega + \Omega)$$

when $\Delta \gg \Omega^{-1}$, we have $(\bar{u}_{\omega\mathbf{k}_\perp}, Ef)_{KG} \approx 0$. Thus Ef is approximately a positive-frequency solution, i.e., $KEf \approx Ef$. An analogous reasoning can be used to show that $E\bar{f}$ is a negative-frequency solution, i.e., $KE\bar{f} \approx 0$. Analogously to Eq. (A6), we have

$$(u_{\omega\mathbf{k}_\perp}, Ef)_{KG} = 2i\epsilon\gamma_{\omega\mathbf{k}_\perp} \frac{\sin[(\omega - \Omega)\Delta/2]}{(\omega - \Omega)}. \quad (\text{A7})$$

Now, by using Eqs. (3.11) and (A3), we write

$$\begin{aligned} \nu^2 &\equiv ||\lambda||^2 = ||KEf||^2 \\ &= \int_0^\infty d\omega \int d\mathbf{k}_\perp |(u_{\omega\mathbf{k}_\perp}, Ef)_{KG}|^2 \\ &\approx 2\pi\epsilon^2\Delta \int d\mathbf{k}_\perp |\gamma_{\Omega\mathbf{k}_\perp}|^2. \end{aligned} \quad (\text{A8})$$

In the particular case where we have a point detector, $\psi(\mathbf{x}) \rightarrow \delta(\mathbf{x})$, we end up with

$$\nu^2 = \frac{\epsilon^2\Omega\Delta}{2\pi}. \quad (\text{A9})$$

For small but not point detectors, let us calculate ν assuming

$$\psi(\mathbf{x}) = \frac{e^{-\mathbf{x}^2/2\kappa^2}}{(\kappa\sqrt{2\pi})^3}$$

in the inertial case, where $\kappa = \text{const}$ is the Gaussian variance. Then,

$$\begin{aligned} \nu_{\text{in}}^2 &= \int d\mathbf{k} |(v_{\mathbf{k}}, Ef)_{KG}|^2 \\ &\approx \frac{\epsilon^2}{4\pi} \int d\mathbf{k} \frac{\delta(\omega_{\mathbf{k}} - \Omega)}{\omega_{\mathbf{k}}} \frac{\sin[(\omega_{\mathbf{k}} - \Omega)\Delta/2]}{(\omega_{\mathbf{k}} - \Omega)} |\hat{\psi}(-\mathbf{k})|^2 \end{aligned}$$

with $v_{\mathbf{k}} = e^{i(\mathbf{k}\mathbf{x} - \omega t)}/\sqrt{16\pi^3\omega_{\mathbf{k}}}$ being positive-frequency Minkowski modes and $\hat{\psi}(\mathbf{k})$ the Fourier transform of $\psi(\mathbf{x})$. Finally, by using $\omega_{\mathbf{k}} = |\mathbf{k}|$ and integrating in spherical coordinates we find

$$\nu_{\text{in}}^2 = \frac{\epsilon^2\Omega\Delta}{2\pi} e^{-\Omega^2\kappa^2}. \quad (\text{A10})$$

Because in the point detector case, $\kappa = 0$, Eqs. (A9) and (A10) are identical, we shall use Eq. (A10) as an approximation for Eq. (A8) associated with the accelerated case provided that $\kappa \ll 1$. This drives us to Eq. (3.13).

-
- [1] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
 - [2] P. M. Alsing, and G. J. Milburn, Phys. Rev. Lett. **91**, 180404 (2003).
 - [3] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
 - [4] L. C. B. Crispino, A. Higuchi and G. E. A. Matsas, Rev. Mod. Phys. **80**, 787 (2008).
 - [5] R. Schutzhold and W. G. Unruh, *Comment on "teleportation with a uniformly accelerated partner"*, quant-ph/0506028.
 - [6] M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, and L. Davidovich, Science **316**, 579 (2007).
 - [7] J. Doukas and L. C. L. Hollenberg, Phys. Rev. A **79**, 052109 (2009).
 - [8] W. G. Unruh and R. M. Wald, Phys. Rev. D **29**, 1047 (1984).
 - [9] We use here the abstract index notation, where latin indices indicate the rank of the four-dimensional space-time tensor, while greek indices express the corresponding components in some particular coordinate system.

- See for example, R. Penrose and W. Rindler, *Spinors and Space-Time: Two-Spinor Calculus and Relativistic Fields* (Cambridge University Press, Cambridge, 1987).
- [10] A. Higuchi, G. E. A. Matsas and C. B. Peres, Phys. Rev. D **48**, 3731 (1993).
- [11] P. Kok and U. Yurtsever, Phys. Rev. D **68**, 085006 (2003).
- [12] R. M. Wald, *Quantum Field Theory in Curved Spacetimes and Black Hole Thermodynamics* (The University of Chicago Press, Chicago, 1994).
- [13] J. Audretsch, *Entangled Systems, New Directions in Quantum Physics* (Wiley, Weinheim, 2007).
- [14] M. A. Nielsen and I. L. Chuang *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
- [15] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
- [16] A. Peres and D. R. Terno, Rev. Mod. Phys. **76**, 93 (2004).
- [17] A. Peres, P. F. Scudo and D. R. Terno, Phys. Rev. Lett. **88**, 230402 (2002).
- [18] R. M. Gingrich and C. Adami, Phys. Rev. Lett. **89**, 270402 (2002).
- [19] A. G. S. Landulfo and G. E. A. Matsas, Phys. Rev. A

- 79**, 044103 (2009).
- [20] G. E. A. Matsas and D. A. T. Vanzella, Phys. Rev. D **59**, 094004 (1999); D. A. T. Vanzella and G. E. A. Matsas, Phys. Rev. Lett. **87**, 151301 (2001); H. Suzuki and K. Yamada, Phys. Rev. D **67**, 065002 (2003).
- [21] R. M. Wald, *Vacuum States in Space-Times with Killing*

Horizons in Quantum Mechanics in Curved Space-Time, NATO Advanced Study Institutes Series, Vol. B230, eds. J. Audrethsch and V. de Sabbata (D. Reidel Publishing Co, Dordrecht, 1990).